

Discrete and Continuous Random Variables



Discrete Random Variables

A discrete random variable X takes a fixed set of possible values with gaps between. The probability distribution of a discrete random variable X lists the values x_i and their probabilities p_i :

Value: $x_1 x_2 x_3 \dots$

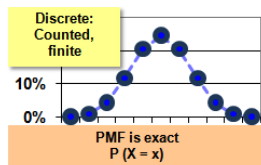
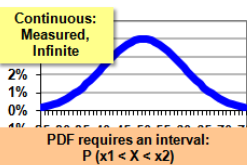
Probability: $p_1 p_2 p_3 \dots$

The probabilities p_i must satisfy two requirements:

1. Every probability p_i is a number between 0 and 1.
2. The sum of the probabilities is 1.

Random Variables

- Usually written as X
- A variable whose possible values are numerical outcomes of a random phenomenon.
- There are two types of random variables, **discrete** and **continuous**.



Expected Value (Mean) of a Discrete Random Variable

The mean of any discrete random variable is an average of the possible outcomes, with each outcome weighted by its probability.

To find the **mean (expected value)** of X , multiply each possible value by its probability, then add all the products

$$\begin{aligned}\mu_x &= E(X) = x_1 p_1 + x_2 p_2 + x_3 p_3 + \dots \\ &= \sum x_i p_i\end{aligned}$$

Discrete Random Variables

- A discrete random variable is one which may take on only a countable number of distinct values such as 0, 1, 2, 3, 4,....
- Discrete random variables are usually (but not necessarily) counts.
- **Examples:**
 - number of children in a family
 - the Friday night attendance at a cinema
 - the number of patients a doctor sees in one day
 - the number of defective light bulbs in a box of ten.

Standard Deviation of a Discrete Random Variable

The **variance** of X is

$$\begin{aligned}Var(X) &= \sigma_x^2 = (x_1 - \mu_x)^2 p_1 + (x_2 - \mu_x)^2 p_2 + (x_3 - \mu_x)^2 p_3 + \dots \\ &= \sum (x_i - \mu_x)^2 p_i\end{aligned}$$

And to find the standard deviation, take the square root of the variance.

$$\text{Standard Deviation} = \sqrt{\text{variance}}$$

Expected Value & Standard Deviation on Calculator

- Enter data (list 1) and probability/frequency (list 2). Probabilities must be entered as decimals.
- Calculate using one-variable statistics and select list 2 as frequency.

L1	L2	L3	1
0	.001		
1	.006		
2	.007		
3	.008		
4	.012		
5	.02		
6	.038		
L1(x) = 0			

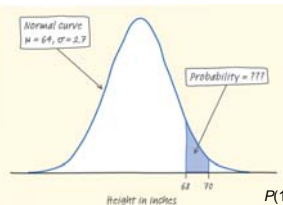
Given that, Y is a continuous random variable whose probability distribution is $N(64, 2.7)$. What is the probability that a randomly chosen young woman has height between 68 and 70 inches?

$$P(68 \leq Y \leq 70) = ???$$

Continuous Random Variable

- A continuous random variable is one which takes an **infinite** number of possible values.
- Continuous random variables are usually measurements.
- Examples:
 - height
 - weight
 - the amount of sugar in an orange
 - the time required to run a mile.

What is the probability that a randomly chosen young woman has height between 68 and 70 inches?



$$P(68 \leq Y \leq 70) = ???$$

$$z = \frac{68 - 64}{2.7} = 1.48 \quad z = \frac{70 - 64}{2.7} = 2.22$$

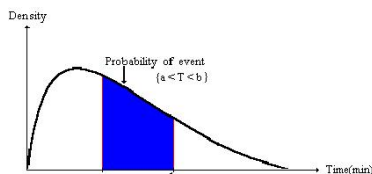
$$P(1.48 \leq Z \leq 2.22) = P(Z \leq 2.22) - P(Z \leq 1.48) = 0.9868 - 0.9306 = 0.0562$$

There is about a 5.6% chance that a randomly chosen young woman has a height between 68 and 70 inches.

Continuous Random Variables

A continuous random variable is not defined at specific values. Instead, it is defined over an interval of value.

You can calculate the probability of a range of values. It is very similar to z-scores and normal distribution calculations.



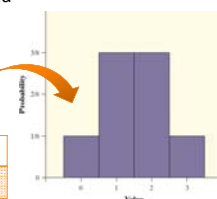
Probability Distribution

The **probability distribution** of a discrete random variable is a list of probabilities associated with each of its possible values. It usually displayed as a chart or graph.

Example: Consider tossing a fair coin 3 times. Define X = the number of heads obtained

- X = 0: TTT
- X = 1: HTT THT TTH
- X = 2: HHT HTH THH
- X = 3: HHH

Value	0	1	2	3
Probability	1/8	3/8	3/8	1/8



Describing the (Probability) Distribution

When analyzing discrete random variables, we'll follow the same strategy we used with quantitative data – describe the shape, center (mean), and spread (standard deviation), and identify any outliers.

